

Graph Theory

Lecture 9: Trees

Faculty Incharge:
Adil Mudasir
Department of CSE, NIT Srinagar

Trees in graph theory

A tree is a connected acyclic graph.

A forest is a graph in which each component is a tree.

Ex.

o

Tree with
one vertex



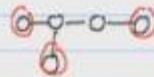
Tree with
two vertices



tree with
three vertices



tree with
4 vertices



tree with 5 vertices

Note: Red nodes are leaf
nodes

leaf of tree is a vertex
of degree one only.

Theorem :

Every Tree with $n(\geq 2)$ vertices has atleast two leaves

A leaf of a tree is a vertex of degree one.

(Th) Every tree with $n(\geq 2)$ vertices has atleast two leaves.

Proof Suppose T has $n(\geq 2)$ vertices.

Let $P : v_0 - v_1 - \dots - v_k$ be the longest path in T .

Since P is longest, every neighbor of v_0 lies on P . Since T is acyclic graph, v_1 is the only neighbor of v_0 . So $d(v_0) = 1$.

Similarly, $d(v_k) = 1$.

So, Tree T has atleast two nodes as leaves if $n \geq 2$ (vertices).
#Hence Proved

Theorem :

Every Tree with n vertices has exactly $n-1$ edges

Theorem Every tree with n vertices has exactly $n-1$ edges.

Proof By induction

Let theorem is true for $n=1$

$n=1, e=0$

Suppose every tree with k vertices has $k-1$ edges, $k \geq 1$.

tree with one vertex

Let T be a tree with $k+1$ vertices.

Since $k+1 \geq 2$, T has a leaf x .

The graph $T-x$, is a tree with k vertices; by induction hypothesis,

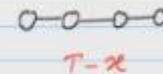
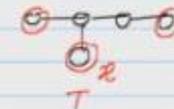
$T-x$ has exactly $(k-1)$ edges.

Since T has one more edge than $T-x$,

the tree T has k edges. \square

Hence Proved

Encircled nodes in red are leaves

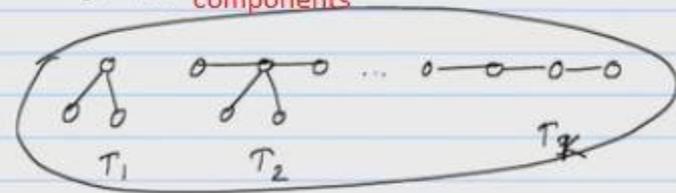


Theorem:

In a forest with v vertices and k components, the number of edges is $v-k$

Theorem In a forest with v vertices and k components, the number of edges is $v-k$.

Proof:



vertices in each component: v_1, v_2, \dots, v_k
edges in each component: $v_1-1, v_2-1, \dots, v_k-1$

$$\begin{aligned} \text{# edges} &= \sum_{i=1}^k (v_i - 1) \\ &= \sum_{i=1}^k v_i - k \\ &= v - k \end{aligned}$$

HENCE proved that:
in a forest with V vertices and K components, no. of edges is $V-K$

Equivalent Statements about Trees

The following statements are equivalent

$$T = (V, E)$$

- (a) T is tree
- (b) T is connected graph and has exactly $|V|-1$ edges
- (c) T is acyclic and has exactly $|V|-1$ edges
- (d) Any two vertices on T are joined by a unique path.

