Graph Theory
Lecture 9: Trees

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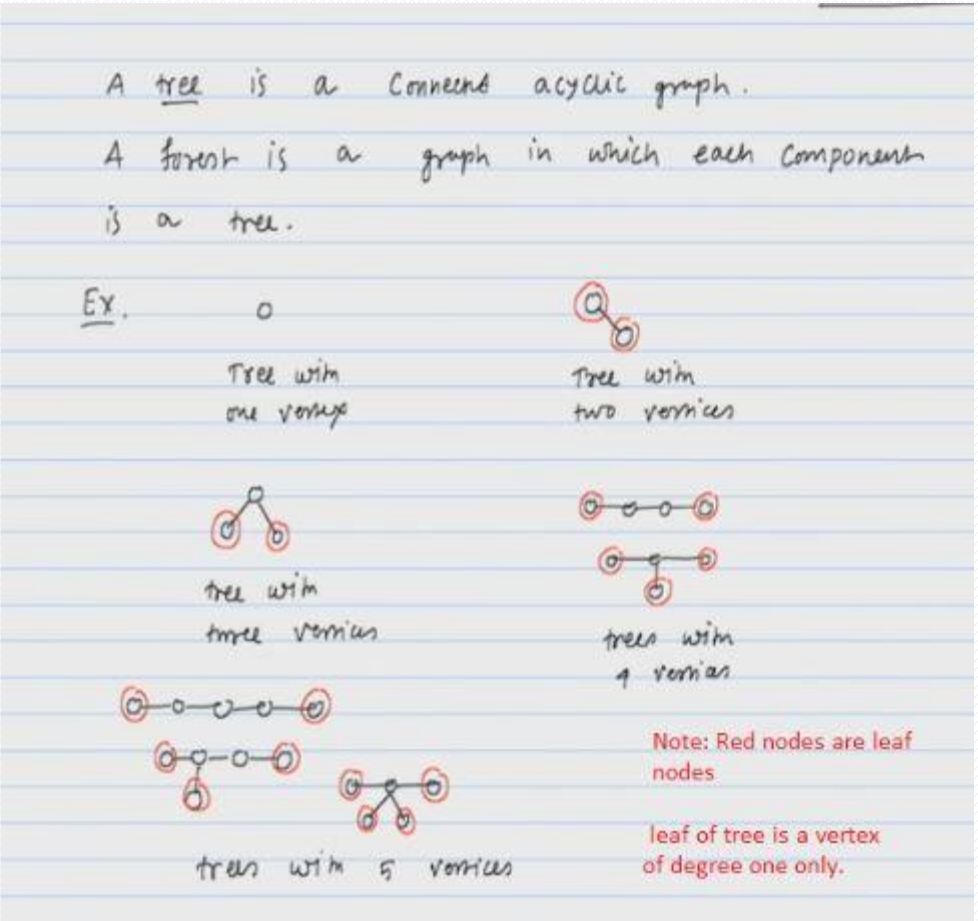
Trees in graph theory



A tree is a connected acyclic graph.

A forest is a graph in which each component is a tree.

Ex.

Theorem :

Every Tree with n(>=2) vertices has atleast two leaves

A leaf of a tree is a vertex of degree one.

Th Every tree with n (≥ 2) vertices has atleast two leaves.

proof Suppose T has $n (\geq 2)$ vertices.

Let $P : v_0 - v_1 - v_2 \cdots v_k$ be the longest path in T.

Since P is longest, every neighbor of v_0 lies on P. Since T is acyclic graph, v_1 is the only neighbor of v_0. So $d(v_0) = 1$.

Similarly, $d(v_k) = 1$.

So, Tree T has atleast two nodes as leaves if n>=2(vertices).
#Hence Proved

Theorem :

Every Tree with n vertices has exactly n-1 edges

Theorem: Every tree with n vertices has exactly $n-1$ edges.

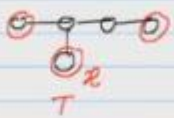
Proof: By induction

Let theorem is true for the $n = 1$ — $n=1$, $e=0$ (tree with one vertex)

Suppose every tree with k vertices has $k-1$ edges, $k \geq 1$.

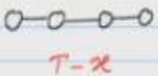
Encircled nodes in red are leaves

Let T be a tree with $k+1$ vertices.



Since $k+1 \geq 2$, T has a leaf x.

The graph $T-x$, is a tree with k vertices; by induction hypothesis, $T-x$ has exactly $(k-1)$ edges.

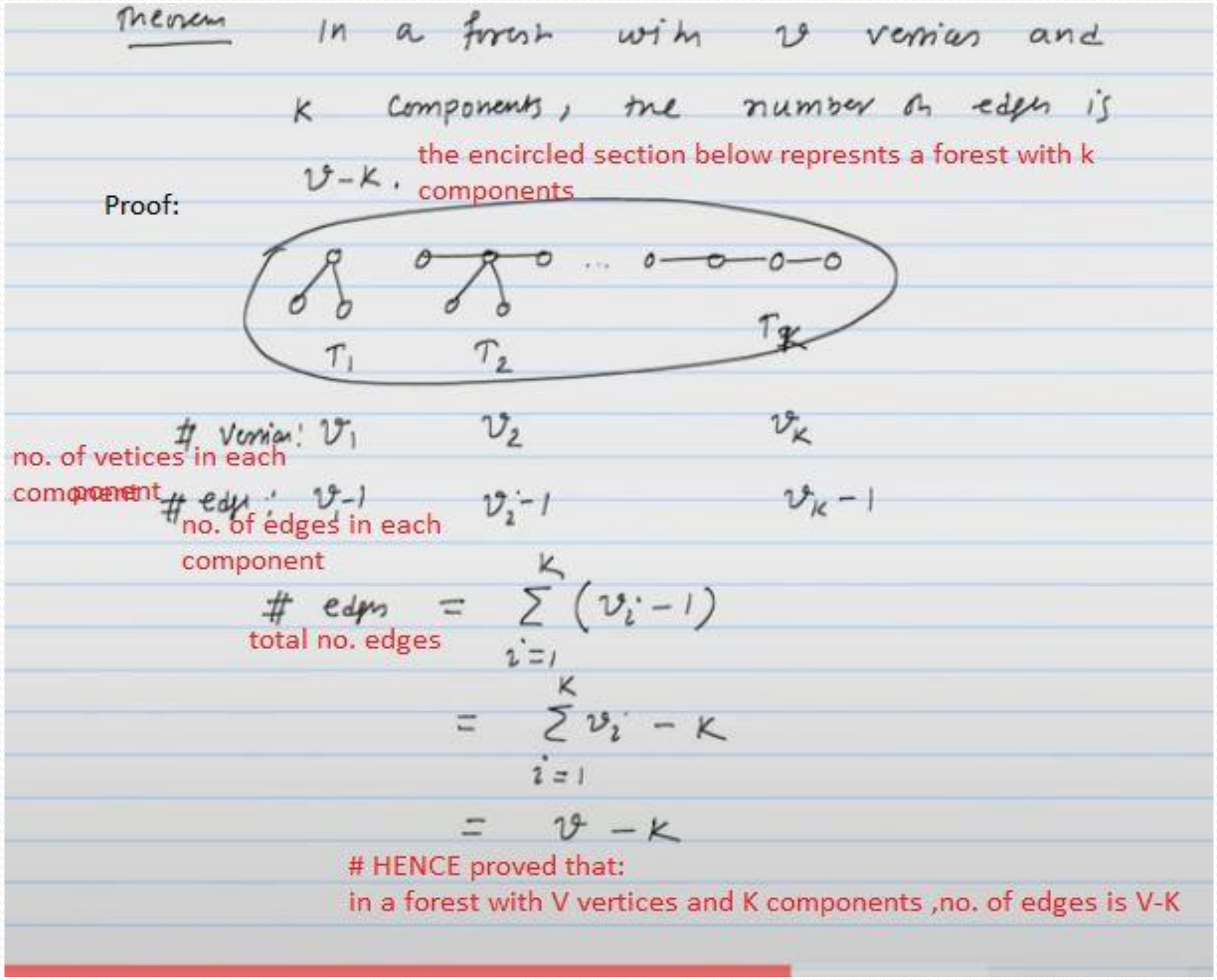


Since T has one more edge than $T-x$, the tree T has k edges. ∎

Hence Proved

Theorem:

In a forest with v vertices and k components , the number of edges is v-k



Equivalent Statements about Trees

The following statements are equivalent

$T = (V, E)$

(a) T is tree

(b) T is connected graph and has exactly $|V|-1$ edges

(c) T is acyclic and has exactly $|V|-1$ edges

(d) Any two vertices in T are joined by a unique path.